Method for Analysis of V/STOL Aircraft Ejectors

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A digital computer program is described for aircraft ejector performance analyses. The effects on performance of temperature ratios, pressure ratios, specific-heat ratios, pressure losses, and aircraft forward speed are included. Momentum correction factors are computed within the program, and the method by which these are determined for rectangular ejectors with multiple nozzles is described. Theoretical and experimental results are compared for rectangular ejectors employing hypermixing nozzles and microjet nozzles. It is demonstrated that the predictions of thrust augmentation ratio are within 3% of the test data. The effects on performance of pressure ratio, temperature ratio, nozzle spacing, mixing section length, inlet and diffuser performances, and the degree of mixing achieved are presented. The analysis clearly demonstrates the importance of rapid mixing, and the role that the ARL hypermixing nozzle has played in advancing the state-of-the-art of aircraft ejectors.

Nomenclature

A	= cross-sectional area
b	= jet half-width
D_h	= mixing section hydraulic diameter
f	= friction factor
h	= mixing section width
J	= jet momentum
K_I	= inlet loss coefficient
K_m	= momentum correction factor
L	= mixing section length
$\dot{m}$	= mass flow rate
N_s	= number of primary nozzles
P	= total pressure
p	= static pressure
S_s	= nozzle spacing
T	= total temperature
t	= static temperature
u	= jet profile velocity
Δu	= excess velocity
v	= one-dimensional velocity
x	= distance downstream of nozzle exits
y	= distance normal to axis of symmetry
γ	= specific heat ratio
ξ	= velocity profile similarity parameter
ξ_f	= nondimensional nozzle spacing
η_D	= diffuser efficiency
ρ	= density

Subscripts

∞	= freestream
0	= primary stream conditions leaving the nozzles
1	= inlet
2	= secondary flow conditions in the nozzle exit plane
3	= diffuser entrance
4	= diffuser exit
e	= secondary streamwake velocity conditions

Introduction

THIS paper presents a theoretical method for the prediction of V/STOL aircraft ejector performance. The theory is applicable to subsonic and choked primary jets, and includes the effects on performance of primary to secondary stream pressure ratio and temperature ratio. The effects of diffusers, inlet pressure loss, and wall friction are also included in the analysis. Application of the quasi-one-dimensional fluid dynamic equations is inadequate for ejector performance predictions unless consideration is given to the shape of the velocity profile at the end of the mixing section. One approach,[1] which has resulted in acceptable ejector performance predictions, is to apply a momentum correction factor K_m to the one-dimensional momentum equation. Values of K_m which have been determined experimentally for cylindrical ejectors, are presented in Ref. 2, but no similar data exists for rectangular ejector configurations.

One of the first analyses of the performance of thrust augmenting ejectors was presented by von Karman.[3] His approach, which concentrated on explaining the performance of Coanda ejectors, analyzed the effects of inlet velocity distribution, but assumed that complete mixing was achieved within the constant area mixing tube. Von Ohain and Campbell[4] developed ejector performance equations which included the effects of component pressure losses and diffuser performance. They demonstrated that the secondary to primary velocity ratio could be expressed explicitly by the solution of a quadratic equation.

The method developed by Von Ohain and Campbell is employed in the present analysis to compute the first approximation for the secondary mass flow. The momentum correction factor is then computed by use of classical two-dimensional jet mixing theory, and a new value of secondary flow is calculated from compressible flow equations. The method employed for the calculation of K_m is essentially an extension of the procedure adopted by Abramovitch[5] for cylindrical ejectors.

Method of Analysis

The ejector stations used in the analysis are defined in Fig. 1. Primary (driving) gas, which may be air or the combustion products of JP4 and air, is injected into the mixing section at station 2 through a series of primary nozzles whose total geometric area is A_p. Nozzle velocity and contraction coefficients are included in the anlysis, and the primary-stream conditions leaving the nozzles are identified by the subscript 0. Secondary (driven) air is induced into the ejector from the freestream, which is denoted by subscript ∞, and undergoes an inlet pressure loss to station 1. External drag of the primary nozzles causes a further pressure loss as the secondary air flows from station 1 to station 2. The primary and secondary streams mix between stations 2 and 3, and exhaust to atmosphere after diffusing to station 4. The degree of mixing achieved in the mixing section is dependent on the noz-

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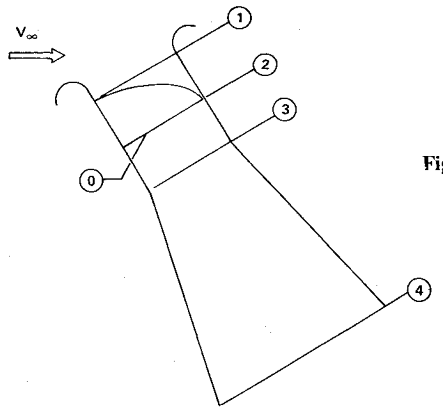

Fig. 1 Ejector stations.

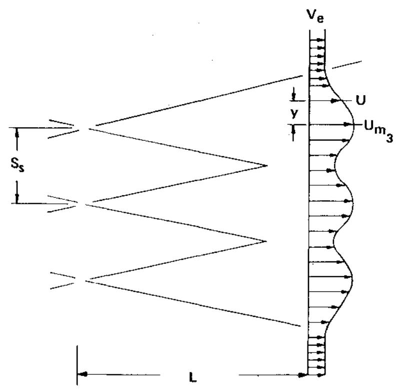

Fig. 2 Downstream velocity profiles.

zle design and spacing, the length of the mixing section, and the thermodynamic and aerodynamic conditions of the two streams at station 2.

One of the objectives of the analytical study was to compare the potential performance of ejectors using slot nozzles and those using hypermixing nozzles. It was necessary, therefore, to develop a theoretical approach to the degree of mixing achievable in the mixing section, which is one of the major parameters affecting ejector performance.

Figure 2 shows the downstream velocity profile created by a series of rectangular nozzles immersed in a secondary stream. The external velocity is denoted v_e, the maximum axial velocity u_m, and the axial velocity at a distance y from the nozzle centerline is u. Assuming similarity of the downstream profiles, we may write

$$(u-v_e)/(u_m-v_e)=f(\xi) \quad -S_s/2\leq y\leq s_s/2 \tag{1}$$

where $\xi=y/b$, b is the half-width of the jet, and S_s is the nozzle spacing. Using Schliciting's[6] expression for self-similar profiles

$$f(\xi)=[1-(y/b)^{1.5}]^2 \tag{2}$$

It is assumed that the nozzles are equally spaced and that the density is constant across the station 3 plane. Defining a new spacing parameter $\xi_f=S_s/2b$, and writing $u_3=v_e+\Delta u_3$ and $u_{m3}=v_e+\Delta u_{m3}$ the stream momentum may be written as

$$J_3=2N_s h\rho_3 b_3\int_0^{\xi_f}[v_e^2+2v_e\Delta u_{m3}f(\xi)+(\Delta u_{m3})^2 f^2(\xi)]\,\mathrm{d}\xi \tag{3}$$

The momentum correction factor K_m which is defined as the ratio of the actual integrated momentum to the bulk average momentum, may then be expressed, after integration, as

$$K_{m3}=(v_e/v_3)(2-v_3)+(1-v_e/v_3)^2F_2(\xi_f)/F_1^2(\xi_f) \tag{4}$$

where

$$F_1(\xi_f)=1-0.8\xi_f^{1.5}+0.25\xi_f^3 \tag{5}$$

and

$$F_2(\xi_f)=1-1.6\xi_f^{1.5}+1.5\xi_f^3-0.7273\xi_f^{4.5}+0.1429\xi_f^6$$

It is now necessary to determine the unmixed secondary-stream velocity v_e, and to relate the spacing parameter ξ_f to known geometric and flow velocity values. The unmixed secondary stream flows essentially isentropically from station 2 to station 3 with density ρ_2. Applying Bernoulli's equation

$$v_e=[v_2^2-2(p_3-p_2)/\rho_2]^{1/2} \tag{6}$$

where p_2 and p_3 are the static pressures at stations 2 and 3.

Calculation of the spacing parameter at station 3, $\xi_f=S_s/2b_3=(A_3/A_0)\,(b_0/b_3)$, requires a knowledge of the jet half-width b_3. This is a function of the distance from the nozzle exits L, the nozzle half-width b_0, the initial jet velocity v_0, the external velocity v_e and the form of the velocity profiles at the nozzle exits and at station 3. The expression for the half-width, which is given by Abramovitch[5] may be simplified considerably by assuming an initial rectangular jet profile and Schlichting's similarity profile at station 3. Under these conditions

$$(b_3/b_0)^2=1+(L/b_0)(v_0/v_e-1)(C/0.45) \tag{7}$$

where the empirical constant C has the value 0.22. Expressing the initial jet half-width as $b_0=(A_0/A_3)(S_s/2)$, and using Eq. (7) results in the following expression for ξ_f:

$$\xi_f=(A_3/A_0)[4.45C(v_0/v_e-1)(L/S_s)(A_3/A_0)+1]^{-1/2} \tag{8}$$

Equations (4-6 and 8) are sufficient to define K_{m3} as a function of ejector geometry, v_0, v_2, ρ_2, p_2, and p_3. Determination of the flow properties cannot be made, however, until the secondary mass flow has been calculated. Since the mass flow depends on many parameters, including K_{m3}, an iterative process is required for a complete solution. A first approximation for the secondary mass flow is provided by an incompressible flow analysis. Applying the pressure-energy equation between stations 1 and 2, the static pressure at station 2 may be expressed as

$$p_2=p_\infty-\tfrac{1}{2}\rho v_I^2(K_I+C_D A_s/A_I)-\tfrac{1}{2}\rho(v_2^2-v_\infty^2)$$

where C_D is the overall nozzle external drag coefficient based on a representative nozzle area A_2, and K_I is the inlet loss coefficient. The pressure loss due to wall friction in the mixing duct is expressed, for convenience, in terms of the dynamic pressure at station 3

$$\Delta p_f=fL\rho v_3^2/2D_h$$

where f is the friction factor and D_h is the hydraulic diameter. In this first approximation, it is assumed that the static pressure at station 4 is equal to the ambient pressure, which, together with the assumption of constant density, allows the pressure at station 3 to be written as

$$p_3=p_\infty-\tfrac{1}{2}\rho\eta_D(v_3^2-v_4^2)$$

The term η_D is the diffuser efficiency. The one-dimensional mixing section momentum equation is

$$\rho A_0v_0^2+\rho A_2v_2^2+p_2A_3=K_m\rho A_3v_3^2+p_3A_3+\Delta p_fA_3 \tag{9}$$

where the momentum at station 3 has been corrected for nonuniform velocity distribution by the momentum correction factor K_m.

Writing $v_4 = (A_0 v_0 + A_2 v_2)\ /A_4$ and eliminating the pressures and density from Eq. (9) results in the quadratic equation

$$\alpha(v_2/v_0)^2 + 2\beta(v_2/v_0) - \gamma = 0 \tag{10}$$

where

$$\alpha = \tfrac{1}{2}[1 + (A_1/A_0 - 1)^2(\psi + 2\Omega - 1)]$$

$$\beta = \Omega(A_1/A_0 - 1)$$

$$\gamma = \tfrac{1}{2}(A_1/A_0)^2(v_\infty/v_0)^2 + (A_1/A_0 - \Omega)$$

$$\psi = K_1 + C_D A_S/A_1$$

$$\Omega = K_m + fL[h + N_s S_s)/4A_1 - \tfrac{1}{2}\eta_D[1 - (A_3/A_4)^2]$$

Solving Eq. (10)

$$v_2/v_0 = (\beta^2 \times \alpha\gamma)^{1/2}/\alpha - \beta/\alpha$$

The mass flow ration is then calculated from

$$(\dot{m}_1/\dot{m}_0)_{\rho=\text{constant}} = (v_2/v_0)(A_2/A_0)$$

Computer Program

The equations just described are used for the initial value of the secondary mass flow. This is then modified in the main program until a balance between the total flow at station 3 and the flow at station 4 is achieved. Since compressible-flow equations are employed, and the specific heat ratio is a function of the static temperature, a series of iterations is required within each overall iteration to compute the conditions at each station.

Freestream conditions are computed, and the first approximation for $\dot{m}_1$ is calculated by the previously described equations. This portion of the program is used only once for each point, and the first value for the density at station 1 is entered into the main program as being equal to ρ_∞. Initial values of the station 1 conditions are computed from the freestream values, the inlet loss coefficient, and the secondary mass flow. A new density is then calculated from the equation of state, and a second approximation to the station 1 conditions is obtained. The iterative process is repeated until convergence is obtained. After calculation of the total pressure at station 2, the Mach number and all the other flow values are computed.

Initial estimates of the density and specific-heat ratio at station 3 are required for the mixed-stream portion of the program. These are provided by mass flow weighted averages. An energy function

$$\mathrm{EF} = (\gamma_0\dot{m}_0 T_0/(\gamma_o - 1) + \gamma_2\dot{m}_2 T_2/(\gamma_2 - 1))/\dot{m}_3$$

a momentum function

$$\mathrm{MF} = (p_0 A_0 + p_2 A_2 + \dot{m}_0 v_0 + \dot{m}_2 v_2)/A_3$$

and a loss function

$$\mathrm{LF} = K_m + fL/2D_h$$

are calculated, from which initial values of the total temperature, velocity, and static pressure are determined by the following equations

$$T_3 = (\gamma_3 - 1)\mathrm{EF}/\gamma_3$$

$$v_3 = \dot{m}_3/A_3\rho_3$$

$$p_3 = \mathrm{MF} - \rho_3 v_3^2 \mathrm{LF}$$

$$T_3/t_3 = \tfrac{1}{2}\{1 + [1 + 2Rg(\gamma_3 - 1)\dot{m}_3^2 T_3/(\gamma_3 A_3^2 p_3^2)]^{1/2}\}$$

The static temperature and density are then computed and a new value of γ_3 is obtained. The iteration proceeds until the static pressure converges.

Diffuser exit total pressure is determined by direct calculation using the equation

$$P_4 = P_3 - (1 - \eta_D)(1 - (A_3/A_4)^2 \rho_3 v_3^2/2$$

Although the diffuser exit total pressure, total temperature, static pressure, and area are known, the exit mass flow cannot be determined until γ_4 has been computed. It is assumed initially that $\gamma_4 = \gamma_3$, and the function $G_4 = (\gamma_4 - 1)/\gamma_4$ is calculated. Then, by use of the isentropic relationship

$$T_4/t_4 = (P_4/p_4)^{G_4}$$

the static temperature, a new γ_4, and a new G_4 are determined. After calculation of the station 4 conditions and the computation of $\dot{m}_4$ from the density, area, and velocity, a comparison is made between the values of $\dot{m}_4$ and $\dot{m}_3$. If they differ by more than $\pm 0.1\%$, the $\dot{m}_4$ value is reduced or increased by one-half of the difference, and $\dot{m}_1$ is set equal to $\dot{m}_4 - \dot{m}_0$. The new $\dot{m}_1$ value is then re-entered into the program and the calculations are repeated. When a mass flow balance has been achieved, it is only the correct solution for the initially assumed value of K_m. A comparison between the values of K_m and the newly computed K_{m_3} is used as the final program branch point. K_m is set equal to K_{m_3} and the new value is re-entered into the program. Finally, when the mass flows and K_m values have converged to the specified accuracies, the program branches to the output section, where thrust, thrust augmentation ratio, mass flow ratio, transfer efficiency, propulsive efficiency, etc., are calculated and printed.

Comparison with Experimental Data

The results of three separate experimental programs have been used to compare the theoretical calculations with test data. These are the ARL static tests, reported by Quinn,[7] in which hypermixing primary nozzles were used; static tests conducted by Ryan,[8] in which struts containing microjets were employed, and the static and wind-tunnel tests reported by Streiff and Henderson.[9] These comparisons demonstrate the flexibility and accuracy of the computational procedure.

Although basic research concerning the performance of hypermixing nozzles is continuing, and an eddy viscosity model for hypermixing jets has been described by Bevilaqua,[10] there is still insufficient information regarding the downstream velofity profiles and the spreading rate for an accurate theoretical prediction of K_m. Tests conducted by ARL,[11] however, indicate that $K_m = 1.01$ is a reasonable value for their configuration. Inlet losses, which included the nozzle

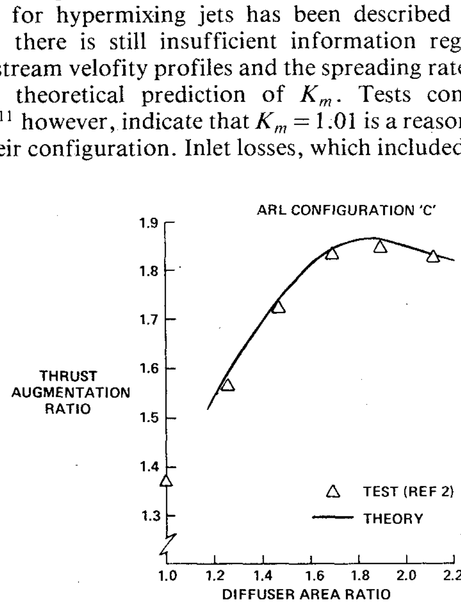

Fig. 3 Comparison of test and theory.

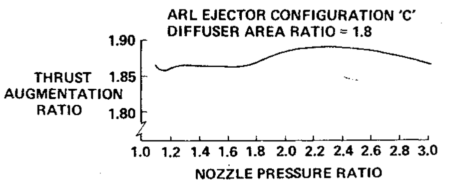

Fig. 4 Effect of nozzle pressure ratio (theory).

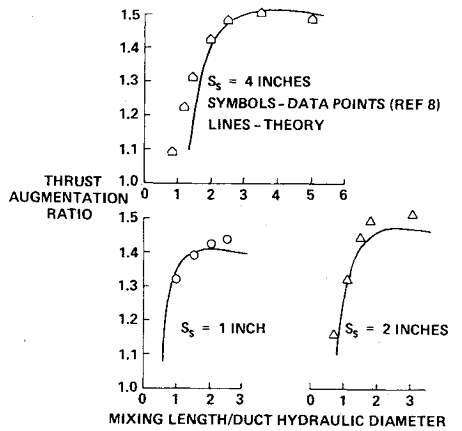

Fig. 5 Comparison of test and theory—microjet nozzles.

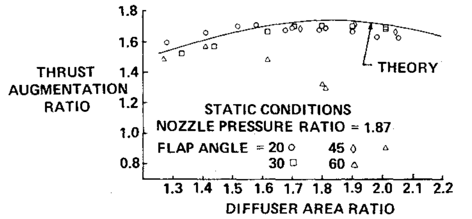

Fig. 6 BAC wing ejector—comparison of test and theory.

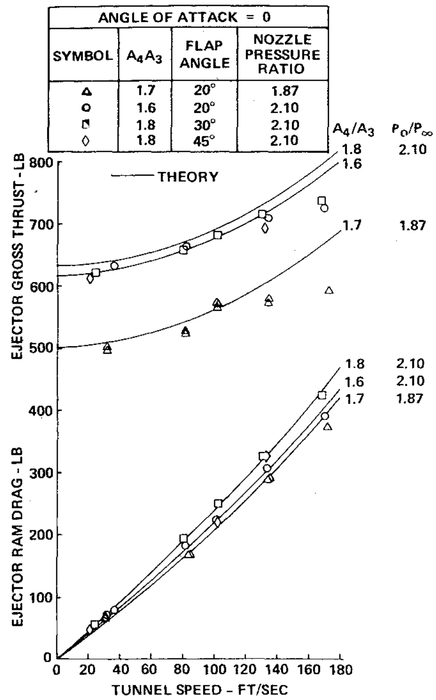

Fig. 7 Comparison of test and theory—Bell wing ejector.

external drag, were also measured by ARL, and it is reported[7] that the inlet loss coefficient = 0.025. Information is also given in Ref. 7 with regard to diffuser performance. The program was used to compute the performance of the ARL ejector configuration C performance. The results, presented in Fig. 3, show excellent agreement with the test data. The effect of primary pressure ratio on thrust augmentation ratio is shown in Fig. 4 for a diffuser area ratio of 1.8. It is seen that ejector performance is essentially independent of pressure ratio in the subsonic primary-flow regime. In the choked regime, however, the augmentation ratio initially rises, reaches a peak at a pressure ratio of 2.3, and then decreases. The variation is nowhere greater than 1½%.

The results of an experimental program conducted by Ryan[8] are interesting because hypermixing nozzles were not used, and the analysis required the calculation of K_{m3}. Primary air was injected through five-hollow struts which each contained sixty-nine 0.043-in. diam orifices located along the trailing edge. The spacing between the orifice centerlines was 0.080 in. The model was constructed such that the strut spacing was adjustable to 1, 2, and 4 in. and the mixing length could be varied from 5.3 to 44.4 in. Test data indicated that the inlet loss coefficient was 0.03, and the nozzle discharge coefficient was 0.904. Ryan's computed strut drag coefficient agreed well with wind-tunnel data measured for the same struts by Krishnamoorthy and Bossler,[12] i.e., $C_D = 0.035$, based on a total strut reference area of 0.333 ft^2.

Experimental and theoretical analyses conducted by Knystautas[13] demonstrated that the velocity profiles resulting from a line of individual circular jets are indistinguishable from those produced by a slot jet after a certain distance x_{2D} downstream of the origin. This distance is dependent on the individual hole diameter d, the spacing between the holes s, and an arbitrarily prescribed amplitude ϵ for which the undulations in the velocity profiles between the individual jets may be considered as having effectively vanished. Denoting the maximum and minimum mean longitudinal velocities in the jet at station x_{2D} by $u_{\max}$ and $u_{\min}$, $\epsilon = (u_{\max} - u_{\min})/u_{\max}$. For the purpose of the following discussion ϵ was selected as 1.0×10^{-4}, implying that undulations of less than 1 in 10,000 are considered insignificant. An analysis of Knystautas' results for this value of ϵ led to the following simple expression for the location of the start of the effectively two-dimensional jet: $x_{2D}/d = 13.33\ (s/d)$.

It is seen from this that the jets issuing from Ryan's strut nozzles were effectively two dimensional 1.06 in. downstream of the strut trailing edges. Since the shortest mixing length tested was five times this distance, it was assumed that the analysis previously described could be applied to Ryan's ejector.

Theoretical and measured thrust augmentation values are shown in Fig. 5 as a function of L/D_h for the three nozzle spacings. The agreement is generally better than 3%, except for the combination of the largest nozzle spacing and shortest mixing length.

The theoretical and experimental static thrust augmentation ratio results for the Bell wing ejector[9] are presented in Fig. 6. Again, the agreement is good, except for the 60° flap setting where the diffuser was badly stalled. Of greater interest, perhaps, is the forward speed performance plotted in Fig. 7. Shown here are the gross thrust and ram drag variations with tunnel speed, for three diffuser area ratios. The variation of inlet loss coefficient with inlet velocity ratio used in these calculations was taken from Rolls-Royce data for lift engine inlets. The predicted thrust and ram drag are seen to be in good agreement with the test data up to a speed of 100 fps. Beyond this tunnel speed the theoretical curves and the test data rapidly diverge, which is probably due to inlet separation.

Additional Performance Predictions

Additional performance predictions, which have not been confirmed by experimental data, are presented in Figs. 8 and

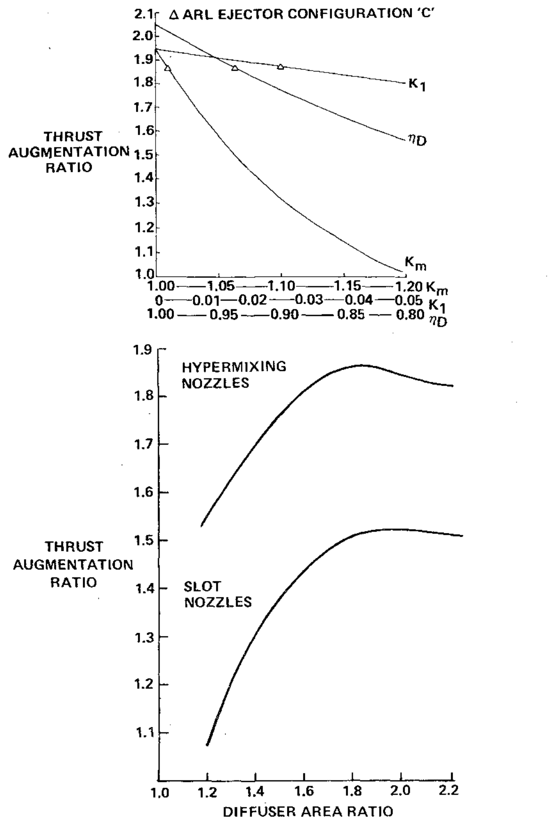

Fig. 8 Effect of mixing, inlet, and diffuser performance on ejector thrust augmentation ratio (theory).

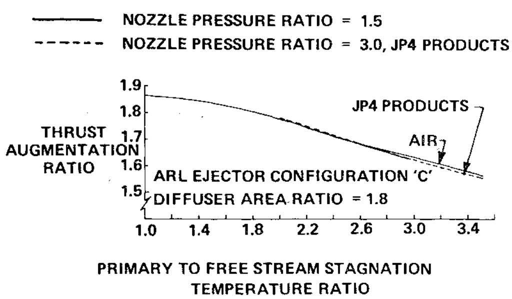

Fig. 9 Effect of temperature ratio (theory).

9. Predictions of the effects of component performance are presented in Fig. 8. ARL ejector configuration C with a diffuser area ratio of 1.8 was selected as the baseline case. Component performance parameters K_I, K_m, and η_D were varied individually, with all other input values constant and equal to their baseline values. The inlet loss coefficient K_I was varied from 0 to 0.05, the nominal value being 0.025. The K_m range was 1.0 to 1.2 (nominal 1.01), and η_D was varied from 1.0 to 0.80 (nominal 0.937). The K_m curve shows the effect of incomplete mixing on ejector performance. For example, an increase in K_m from 1.01 to 1.02 causes a reduction in augmentation ratio from 1.865 to 1.790, a drop of 4%. Figure 8 also shows the degradation in thrust augmentation ratio that would result if the hypermixing nozzles installed in the ARL ejector configuration C were replaced by slot nozzles. The same diffuser performance was assumed for these calculations, although it was realized that it would most probably be lower, due to the nonuniform velocity profile at the end of the mixing section. All geometric parameters and flow properties were assumed to be the same in the calculation of these curves, the only difference being that the hypermixing nozzles were replaced by slot nozzles. The role of ARL hypermixing nozzles in advancing the state-of-the-art of ejectors for thrust augmentation is clearly demonstrated by these curves. The effect of increasing primary-stream temperature is given in Fig. 9. It is seen that the performance is almost independent of nozzle pressure ratio and whether air or the combustion products of JP4 and air are used. It might be argued that the higher temperatures would promote more rapid mixing, which is true for slot nozzles, but K_{m3} was assumed to be 1.01 throughout these computations, which is equivalent to assuming that the flows were almost completely mixed.

Conclusions

A theory has been presented for the performance of rectangular ejectors employing slot nozzles, and an ejector performance computer program has been developed. The method has been shown to predict static thrust augmentation ratio to within 3% of test data. It has also been demonstrated that STOL performance may be predicted accurately if no flow separations occur within the ejector. Augmentation ratio has been shown to be almost independent of primary nozzle pressure ratio, but an increase in nozzle temperature ratio causes a performance degradation. The importance of achieving rapid mixing of the primary and secondary streams has been demonstrated.

References

[1]London, A., "Exhaust-Stack Ejectors for Marine Gas Turbine Installations," Stanford University Tech. Rept. 26, 1955.

[2]*Aerospace Applied Thermodynamics Manual,* SAE, Warrendale, Pa., 1960.

[3]Von Karman, T., "Theoretical Remarks on Thrust Augmentation," Reissner Anniversary Volume, Edwards Brothers, Ann Arbor, Mich., 1949, pp. 461-468.

[4]Von Ohain, H. and Campbell, S., "Thrust Augmentation for V/STOL, ARL's Research and Concepts," ARL Rept. 67-0065, March 1967.

[5]Abramovitch, G., *The Theory of Turbulent Jets,* MIT Press, Cambridge, Mass., 1963.

[6]Schlichting, H., *Boundary Layer Theory,* McGraw-Hill, New York, 1962.

[7]Quinn, B., "Recent Developments in Large Area Ratio Thrust Augmentors," *Journal of Aircraft,* Vol. 10, Aug. 1973, pp. 481-486.

[8]Ryan, E., "An Experimental Evaluation of Jet Mixing Characteristics and Performance of a Rectangular Ejector Employing Quiet Microjet Primary Nozzles," M.S. Thesis, Univ. of New York at Buffalo, Sept. 1973.

[9]Streiff, H. and Henderson, C., "Ejector Thrust Augmentation for STOL Aircraft Applications," AIAA/SAE 10th Propulsion Conference, San Diego, Calif., Oct. 21-24, 1974.

[10]Bevilaqua, P., "An Eddy Viscosity Model for Hypermixing Jets and Wakes," ARL Rept. 72-0047, April 1972.

[11]Bevilaqua, P. and Toms, H., "A Comparison Test of the Hypermixing Nozzle," ARL TR 74-0006, Jan. 1974.

[12]Krishnamoorthy, V. and Bossler, F. B. , "Jet Noise Reduction for Military Reconaissance/Surveillance Aircraft," Bell Aerospace Co. Rept. 7389-945005A, Phase 1, Task 5, April 1972.

[13]Knystautas, R., "The Turbulent Jet from a Series of Holes in Line," *Aeronautical Quarterly,* Vol. XV, Feb. 1964.